Computers in Biology and Medicine 38 (2008) 583–600

Computers in Biology and Medicine

www.intl.elsevierhealth.com/journals/cobm

Statistical correlations and risk analyses techniques for a diving dual phase bubble model and data bank using massively parallel supercomputers

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Received 29 May 2007; accepted 12 February 2008

Abstract

Linking model and data, we detail the LANL diving reduced gradient bubble model (RGBM), dynamical principles, and correlation with data in the LANL Data Bank. Table, profile, and meter risks are obtained from likelihood analysis and quoted for air, nitrox, helitrox no-decompression time limits, repetitive dive tables, and selected mixed gas and repetitive profiles. Application analyses include the EXPLORER decompression meter algorithm, NAUI tables, University of Wisconsin Seafood Diver tables, comparative NAUI, PADI, Oceanic NDLs and repetitive dives, comparative nitrogen and helium mixed gas risks, USS Perry deep rebreather (RB) exploration dive,world record open circuit (OC) dive, and Woodville Karst Plain Project (WKPP) extreme cave exploration profiles. The algorithm has seen extensive and utilitarian application in mixed gas diving, both in recreational and technical sectors, and forms the bases forreleased tables and decompression meters used by scientific, commercial, and research divers. The LANL Data Bank is described, and the methods used to deduce risk are detailed. Risk functions for dissolved gas and bubbles are summarized. Parameters that can be used to estimate profile risk are tallied. To fit data, a modified Levenberg–Marquardt routine is employed with $L2$ error norm. Appendices sketch the numerical methods, and list reports from field testing for (real) mixed gas diving. A Monte Carlo-like sampling scheme for fast numerical analysis of the data is also detailed, as a coupled variance reduction technique and additional check on the canonical approach to estimating diving risk. The method suggests alternatives to the canonical approach. This work represents a first time correlation effort linking a dynamical bubble model with deep stop data. Supercomputing resources are requisite to connect model and data in application.
Published by Elsevier Ltd.

Keywords: Bubble model; Diving risk; LANL Data Bank; Decompression diving; Maximum likelihood; Monte Carlo sampling

1. Introduction

For coupled model and data correlation, we detail the LANL reduced gradient bubble model (RGBM), dynamical principles, and correlation with profiles in the LANL Data Bank. Table, meter, and profile risks deduced in likelihood analysis are indicated along with risks parameters. Application analyses include the EXPLORER decompression meter algorithm, NAUI Tables, University of Wisconsin Seafood Diver Tables, comparative NAUI, PADI, Oceanic NDLs and repetitive dives, comparative nitrogen and helium mixed gas risks, USS Perry deep RB exploration dive, world record open circuit (OC) dive, and Woodville Karst Plain Project (WKPP) extreme cave exploration profiles. The model enjoys extensive and utilitarian application in mixed gas diving, both in recreational and technical sectors, and forms the bases for released tables and decompression meters used by scientific, commercial, and research divers. Supercomputing power is necessary for application and correlation of model and data.

Our intent here is to also cover aspects of the RGBM not detailed in earlier publications. To do this, we have been collecting mixed gas, deep stop, decompression data in the technical diving arena. This is necessary for model and data correlation, that is, most existing data is based on the shallow stop paradigm required by dissolved gas models, thus biased versus deep stop staging. Deep stop data is needed today, within the RGBM,

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0010-4825/$ - see front matter Published by Elsevier Ltd.
doi:10.1016/j.compbiomed.2008.02.006

as well as all other bubble (dual phase) models requiring deep stops algorithmically. While our data is broadbased, we have been able to deduce some correlation parameters, plus estimate some table, meter, and profile risks. Data collection continues across the gamut of technical, scientific, and research diving.

It is noted that the so-called diving units are employed herein, that is, standard SI units for depth and pressure are not used. Pressures and depths are measured in feet-of-seawater (fsw) because of widespread usage in the diving community. The conversion is simple,

$$33.28\,\text{fsw} = 1\,\text{atm}.$$

In the metric sense, meters-of-seawater (msw) is also employed,

$$10\,\text{msw} = 33.28\,\text{fsw}.$$

Breathing mixtures, such as nitrox (nitrogen and oxygen), heliox (helium and oxygen), and trimix (helium, nitrogen, and oxygen), carry standardized notation. If the fraction of oxygen is greater than 21%, the mixture is termed enriched. Enriched nitrox mixtures are denoted EANx, enriched heliox mixtures are denoted EAHx, and enriched trimix mixtures are denoted EATx, for x the oxygen percentage. For other mixtures of nitrox and heliox the convention is to name them with inert gas percentage first, and then oxygen percentage, such as 85/15 nitrox or 85/15 heliox. For trimix, notation is shortened to list the oxygen percentage first, and then only the helium percentage, such as, 15/45 trimix, meaning 15% oxygen, 45% helium, and 40% nitrogen. Air is interchangeably denoted EAN21 or 79/21 nitrox.

2. Overview

The systematics of gas exchange [1–6], nucleation [2,7–12, 82], bubble growth [1,12–16] and elimination [17–21], counterdiffusion [4,22–25], oxygen impact [26–32], and adaptation [9,17,33–38] upon diving decompression staging [3,18,28,29,35,36,39–58,87,90], and attendant altitude modifications [3,27,46,56,59,60] are so complicated that theories only reflect pieces of the puzzle. Computational algorithms, tables, and manned testing are, however, requisite across a spectrum of activities. And the potential of electronic devices to process tables of information or detailed equations underwater is near maturity, with virtually any algorithm amenable to digital implementation. Pressures for even more sophisticated algorithms are expected to grow.

Still computational models enjoy varying degrees of success or failure. More complex models address a greater number of issues, but are harder to codify in decompression tables. Simpler models are easier to codify, but are less comprehensive. Some models are based on first principles, but most are not. Application of models can be subjective in the absence of definitive data, the acquisition of which is tedious, sometimes controversial, and often ambiguous. If deterministic models are abandoned, statistical analysis can address the variability of outcome inherent to random occurrences, but only in manner indifferent to specification of controlling mechanisms. The so called dose–response characteristics of statistical analysis are very attractive in the formulation of risk tables [55,56,59,61,62]. Applied to decompression sickness (DCS) incidence, tables of comparative risk offer a means of weighing contributing factors and exposure alternatives. At the basis of statistical and probabilistic analyses of DCS is the binomial distribution. The binomial distribution is the fundamental frequency distribution governing random events, like DCS [63,64].

With coupled model and data correlation, we detail the RGBM [6,60,65,66] on dynamical principles, and then its statistical correlations. Both dissolved gas and bubble risk functions are described and parameterized from data in the LANL Data Bank. The RGBM uses a bubble volume to limit exposures, not critical tensions. Bubble volumes are estimates of separated gas phases, and the limit point is called the phase volume. Critical tensions are limit points to dissolved gas buildup in arbitrary tissue compartments, and are often called M-values. The approach is computationally iterative, and though mathematically intensive, diving microprocessors today easily handle calculations in the millisecond processing time frame. The algorithm is the basis of released mixed gas technical tables (NAUI Technical Diving, Tampa, 2002) and simplified recreational air and nitrox tables up to 10,000 ft elevation. Meter implementations of the RGBM are available and under continuing development, specifically HydroSpace, Zeagle, Steam Machines, Underwater Technologies, Mares, Dacor, Suunto, Plexus, and other players. Commercial RGBM software includes GAP, ABYSS, and HydroSpace EXPLORER Simulator. All have exhibited safe and efficient operation from the diving perspective, as will be detailed.

3. RGBM synthesis

The RGBM employs a phase volume [58,67,68,88–90] constraint across the dive profile, tracking excited bubble volumes over the dive. Bubble structures are represented by flexible seed skins with appropriate material properties, permeable to gas diffusion at all pressures and temperatures. Gas diffuses across the bubble interface, and the bubble is subject to Boyle expansion–contraction. The phase volume is an estimate of the cumulative volume of bubbles left at the surface after arbitrary depth–time exposures on any diving breathing mixture. Mixtures can be nitrox (oxygen and nitrogen, including air), heliox (oxygen and helium), and trimix (oxygen, helium, and nitrogen). These bubbles can expand and contract during the dive, and are assumed to be excited off an exponential distribution that decreases in number as the radius of the excited bubbles increases. The material properties of these bubbles determine their response to pressure changes, inert gas (nitrogen and helium) diffusion across their interfaces, and the excitation radii for growth. Collectively, material properties are tabulated within equations-of-state (EOS) [83] for lipid and aqueous bubble coatings [86,91].

The phase volume constraint equation is written in terms of a phase function, $\dot{\phi}$, varying in time, for τ_{ex} the bubble excitation time, across a distribution of excited bubble seeds, n,

$$\int_{\tau_{ex}}^{\tau} \frac{\partial \phi}{\partial t}\, dt \leqslant \Phi$$

with tagging the three bubble processes of excitation, interface gas diffusion, and Boyle expansion–contraction,

$$\dot{\phi} = \frac{\partial \phi}{\partial t}$$

for Φ the separated phase, and τ some (long) cutoff time. More particularly, for Π the total gas tension, taking $\tau \to \infty$, with V the separated phase volume, P the pressure, and T the temperature,

$$\dot{\phi} = \left[\frac{\partial V}{\partial t}\right]_{\text{diffusion}} + \left[\frac{\partial V}{\partial t}\right]_{\text{Boyle/Charles}} + \left[\frac{\partial V}{\partial t}\right]_{\text{excitation}}$$

for,

$$\left[\frac{\partial V}{\partial t}\right]_{\text{diffusion}} = 4\pi\beta \exp(\beta\varepsilon) DS \int_{\varepsilon}^{\infty} nr \left[\Pi - P - \frac{2\gamma}{r}\right] dr,$$

$$\left[\frac{\partial V}{\partial t}\right]_{\text{Boyle/Charles}} = 4\pi\beta \exp(\beta\varepsilon) \int_{\varepsilon}^{\infty} nr^2 \left[\frac{\partial r}{\partial P}\frac{\partial P}{\partial t} + \frac{\partial r}{\partial T}\frac{\partial T}{\partial t}\right] dr,$$

$$\left[\frac{\partial V}{\partial t}\right]_{\text{excitation}} = 4\pi \frac{\partial}{\partial t}\left[\theta(t-\tau_{ex}) \int_{\varepsilon}^{\infty} nr^2\, dr\right]$$

and

$$n = \exp(-\beta r)$$

with quantities as noted, and seed density, n, normalized to the excited phase volume, V,

$$4\pi \int_{\varepsilon}^{\infty} nr^2\, dr = \exp(-\beta\varepsilon)[8\pi\beta^{-3} + 8\pi\varepsilon\beta^{-2} + 4\pi\varepsilon^2\beta^{-1}] = V$$

for ε the seed excitation radius, r the bubble radius, γ the surface tension, D the diffusivity, S the solubility, and the step (heaviside) function, θ, defined for seed excitation at time, τ_{ex},

$$\theta(t-\tau_{ex}) = 0, \quad t \leqslant \tau_{ex}$$

$$\theta(t-\tau_{ex}) = 1, \quad t > \tau_{ex}$$

with the time derivative of the heaviside function a delta function,

$$\frac{\partial \theta(t-\tau_{ex})}{\partial t} = \delta(t-\tau_{ex}).$$

In the integrals over time, we do not consider flying-after-diving scenarios.

In lowest order, number densities of nitrogen and helium bubble seeds are comparable [16]. Experiments suggest that helium bubbles are smaller but more numerous than nitrogen bubble seeds in the same substrate measurements, but differences are small. In zeroth order,

$$n_{He} \simeq n_{N_2} = n.$$

Table 1
RGBM excitation radii

Pressure P (fsw)	Excitation radius ε (μm)	Pressure P (fsw)	Excitation radius ε (μm)
13	0.174	153	0.033
33	0.097	183	0.029
53	0.073	283	0.024
73	0.059	383	0.016
93	0.051	483	0.011
113	0.046	583	0.009

In higher order, helium and nitrogen seed densities are averaged over breathing mixture fractions, f_{He} and f_{N_2}, for an effective number density, n,

$$n = \frac{f_{He} n_{He} + f_{N_2} n_{N_2}}{f_{He} + f_{N_2}}.$$

The skin EOS quantifies the response of bubble films under changes of pressure, P, and temperature, T. An EOS is complicated, often only tabular, or implicitly defined as function of seed volume. To simplify bubble skin EOS lookups, Boyle factors, ξ, are used, so that,

$$\xi PV = nRT$$

as codified in Table 2. For mixed gas diving, Π is the sum of nitrogen and helium dissolved gas loadings, and the dissolved gradient, G, is,

$$G = \Pi - P.$$

Thus the phase function, $\dot{\phi}$, depends on the number of bubbles, n, stimulated into growth by compression–decompression, the supersaturation gradient, G, seed expansion–contraction by radial diffusion, $\partial r/\partial t$, Boyle expansion–contraction with pressure changes, P, and inside temperature, T, in general. The excitation radius, ε, depends on material properties [8,14,69], and is taken for nitrogen (μm),

$$\varepsilon_{N_2} = 0.007 + 0.016\left[\frac{T}{P}\right]^{1/3} + 0.041\left[\frac{T}{P}\right]^{2/3}$$

and for helium,

$$\varepsilon_{He} = 0.003 + 0.015\left[\frac{T}{P}\right]^{1/3} + 0.025\left[\frac{T}{P}\right]^{2/3}$$

for T measured in absolute K, and P given in fsw, with ranges for virial coefficients, aqueous to lipid material, ξ, s, varying by factors of 0.75–4.86 times the values listed above [2,70]. Both expression above represent fits to RGBM mixed gas data across lipid and aqueous bubble films [8,71,79], and are different from other phase models [72,73]. Values of excitation radii, ε, above range from 0.01 to 0.05 μm for sea level down to 500 fsw. This is compared to excitation radii in other models, varying permeability model [16,58] and tissue bubble diffusion model [72], which vary in the 1 μm range. In the very large pressure limit, excitation radii are in the 1/1000 μm range. Table 1 lists excitation radii (air) according to the RGBM.

Table 2
RGBM Boyle multipliers

Depth (fsw)	EOS multiplier ξ
30	0.610
90	0.732
150	0.859
210	0.939
270	1.032
330	1.119
390	1.169
450	1.183
510	1.203

Table 3
RGBM mass transfer coefficients

Gas	DS $\times 10^{-6}$ (μm^2/s fsw)
H_2	72.5
He	18.4
Ne	10.1
N_2	56.9
Ar	40.7
O_2	41.3

To track Boyle bubble expansion–contraction easily, a set of multipliers, ξ, is tabulated in Table 2 reducing EOS data for just pressure changes. For changes in pressure, we have, for bubble assemblies of volume, V, at ambient pressure, P,

$$\xi_i P_i V_i = \xi_f P_f V_f$$

simply, with i and f denoting initial and final states. Multipliers represent a 50/50 lipid–aqueous skin, following Sears [74] and Blank [13]. These multipliers represent a simplification of extensive EOS data for lipid and aqueous materials, condensed into the simpler pressure–volume form above.

To track gas transfer across bubble boundaries, we need the mass transport coefficients, DS, for inert gases. Table 3 lists DS for the same 50/50 lipid–aqueous surface, using Frenkel [10], Lango [13], and Batchelor [80]. Mass transfer coefficients are just phenomenological diffusion coefficients for complex gas transport across lipid and aqueous bubble surfaces in tissue and blood. They are a combination of measurements and data extrapolation of gas transfer estimates for inert gases.

Notice that helium has a low mass transport coefficient, some three times smaller than nitrogen.

Three parameters, closing the set, are nominally,

$$\Phi = 596. \pm 210\,\mu m^3$$

and, for nitrogen and helium,

$$\beta_{N_2} = 0.684 \pm 0.282\,\mu m^{-1}$$

$$\beta_{He} = 0.573 \pm 0.194\,\mu m^{-1}$$

with

$$2\gamma = \sigma\left[44.7\left(\frac{P}{T}\right)^{1/4} + 24.3\left(\frac{P}{T}\right)^{1/2}\right]\ \text{dyne/cm}$$

with material property, σ,

$$0.10 \leqslant \sigma \leqslant 0.85$$

moving from lipid to watery tissue. Later in this analysis, we take $\sigma = 0.5$. The first two parameter sets were obtained from fitting the algorithm to published no decompression time limits (NDLs) for air, nitrox, trimix, and heliox [31,40,53,57,75]. The third parameter follows from EOS estimates of surface tension, as with excitation radii. Tissues and blood are undersaturated with respect to ambient pressure as far as inert gas partial pressures (tensions). This produces the necessary ingradient for oxygen and outgradient for carbon dioxide in metabolic processes. The difference is termed the inherent undersaturation. The inherent unsaturation (or oxygen window), ψ, takes the form, [3,58] (fsw),

$$\psi = f_{O_2} P - 2.04(1 - f_{O_2}) - 5.47$$

a linear function of oxygen partial pressure up to 2.0 atm and then constant beyond that, near 70 fsw, with P ambient pressure, and f_{O_2} oxygen fraction. Under compression–decompression, some of this window likely takes up inert gases, denoted, ζ,

$$\zeta = f_{O_2} P - \psi$$

and is added to the inert gas tension. In time, it is assumed, for inert gas, k,

$$\zeta_k = \left[\frac{f_k}{1 - f_{O_2}}\right][f_{O_2} P - \psi][1 - \exp(-\lambda_k t)],$$

for λ_k a decay constant, f_{O_2} again the oxygen fraction, and f_k the inert gas mixture fraction (same across all compartments). Inert gas fractions, f_k, plus oxygen fraction, f_{O_2}, sum to 1,

$$f_{O_2} + \sum_{k=1}^{K} f_k = 1,$$

where $K = 2, k = N_2$, He, that is, mixed gas diving. Tissue tensions (partial pressures), p_k, for ambient partial pressure, p_{ak}, and initial tissue tension, p_{ik}, evolve in time, t, in usual fashion in compartment, τ_k, according to, given v the (linear) ascent or descent rate between stages,

$$p_k - p_{ak} + \frac{v}{\lambda_k} = vt + \left[p_{ik} - p_{ak} + \frac{v}{\lambda_k}\right]\exp(-\lambda_k t) + \zeta_k$$

for,

$$\lambda_k = \frac{0.693}{\tau_k}$$

for τ_k tissue halftime, and ambient pressure, P, as a function of depth, d, in units of fsw,

$$P = \eta d + P_h$$

for surface ambient pressure, P_h,

$$P_h = 33\exp(-0.0381h)$$

given h in multiples of 1000 ft elevation, $\eta = 1$ for salt water, and $\eta = 0.975$ for fresh water. For any gas with mixture fraction, f_k, obviously,

$$p_{ak} = f_k P$$

and total tension, Π, is the sum of component tensions,

$$\Pi = \sum_{k=1}^{K} p_k.$$

Nitrogen halftimes, τ_{kN_2}, are taken to be 2.5, 5, 10, 20, 40, 80, 120, 180, 240, 320, and 480 min. Helium halftimes, τ_{kHe}, are 2.65 times faster for the same nitrogen compartments,

$$\tau_{kHe} = \frac{\tau_{kN_2}}{2.65}.$$

The bubble dynamical protocol in the RGBM algorithm amounts to staging on the seed number averaged, free-dissolved gradient across all tissue compartments, G,

$$G \int_{\varepsilon}^{\infty} n \, dr = (\Pi - P) \int_{\varepsilon}^{\infty} n \, dr \leqslant \int_{\varepsilon}^{\infty} \left[\frac{2\gamma}{r} \right] n \, dr,$$

so that

$$G = (\Pi - P) \leqslant \beta \exp(\beta\varepsilon) \int_{\varepsilon}^{\infty} \exp(-\beta r) \left[\frac{2\gamma}{r} \right] dr$$

for ε the excitation radius at P and T. Time spent at each stop is iteratively calculated so that the total separated phase, Φ, is maintained at, or below, its limit point. This requires some computing power, but is attainable in diver wrist computers presently marketed commercially. Stops are computed in 10 fsw increments. An important feature of the iterative process is noted:

1. Separated phase volume, Φ, is the same for all inert gases.
2. The gradient, G, is slowly varying as seeds are excited into growth, expand or contract as gas diffuses across bubble films, and expand or contract as ambient pressure changes.

The combination of the two produces dramatically different staging regimens than classical dissolved gas protocols. This (new) staging protocol has been in use for the past 8–12 years, data are being collected from divers, and the process of evaluation and updating is a continuous one.

4. RGBM profile Data Bank

Divers using bubble models are reporting their profiles to a Data Bank, located at LANL (also NAUI Technical Diving Operations). The profile information requested is simple:

1. bottom mix/pp_{O_2}, depth, and time (square wave equivalent);
2. ascent and descent rates;
3. stage and decompression mix/pp_{O_2}, depths, and times;
4. surface intervals;
5. time to fly;
6. diver age, weight, and sex;
7. outcome (health problems), rated 1–5 in order of poor to well.

This information aids validation and extension of model application space. Some 2823 profiles now reside in the LANL Data Bank. The are 19 cases of DCS in the data file. The underlying DCS incidence rate is, $p = 19/2823 = 0.0067$, below 1%. Stored profiles range from 150 fsw down to 840 fsw, with the majority above 350 fsw. All data enters through the authors (BRW. and TRO.), that is, divers, profiles, and outcomes are filtered. A summary breakdown of DCS hit (bends) data consists of the following:

1. OC deep nitrox reverse profiles—5 hits (3 DCSI, 2 DCSII);
2. OC deep nitrox—3 hits (2 DCSI, 1 DCSII);
3. OC deep trimix reverse profiles—2 hits (1 DCSII, 1 DCSIII);
4. OC deep trimix—2 hits (1 DCSI, 1 DCSIII);
5. OC deep heliox—2 hits (2 DCSII);
6. RB deep nitrox—2 hits (1 DCSI, 1 DCSII);
7. RB deep trimix—1 hit (1 DCSIII);
8. RB deep heliox—2 hits (1 DCSI, 1 DCSII).

DCSI means limb bends, DCSII implies central nervous system (CNS) bends, and DCSIII denotes inner ear bends (occurring mainly on helium mixtures). DCII and DCSIII are fairly serious afflictions, while DCSI is less traumatic. Deep nitrox means a range beyond 150 fsw, deep trimix means a range beyond 200 fsw, and deep heliox means a range beyond 250 fsw as a rough categorization. The abbreviation OC denotes open circuit, while RB denotes rebreather. Reverse profiles are any sequence of dives in which the present dive is deeper than the previous dive. Nitrox means an oxygen enriched nitrogen mixture (including air), trimix denotes a breathing mixture of nitrogen, helium, oxygen, and heliox is a breathing mixture of helium and oxygen. None of the trimix nor heliox cases involved oxygen enriched mixtures on OC, and RB hits did not involve elevated oxygen partial pressures above 1.4 atm. Nitrogen-to-helium (*heavy-to-light*) gas switches occurred in two cases, violating contemporary isobaric counterdiffusion (ICD) protocols [4,22,23,29,47]. ICD refers to two inert gases (usually nitrogen and helium) moving in opposite directions in tissues and blood. When summed, total gas tensions (partial pressures) can lead to increased supersaturation and bubble formation probability. None of the set exhibited full body nor CNS oxygen toxicity. The 19 cases come *after the fact*, that is diver distress, most with, but some without, hyperbaric chamber treatment following distress. Appendix A describes many of the profiles in the RGBM Data Bank, as well as broader field testing reported to us. Profiles come from seasoned divers using wrist slate decompression tables with computer backups. Some profiles come to us directly as computer downloads, which we transcribe to the requisite format.

Profiles come from the technical diving community at large, essentially mixed gas, extended range, decompression, and extreme diving. Profiles from the recreational community are not included, unless they involve extreme exposures on air or nitrox (many repetitive dives, deeper than 150 fsw, altitude exposures, etc.). This low rate makes statistical analysis difficult, and we use a global approach to defining risk after we fit the model to the data using maximum likelihood. The maximum likelihood fit links directly to the binomial probability structure of DCS incidence in divers and aviators. Consider it briefly, and the likelihood maximization technique [63,64,76].

5. Probabilistics

DCS is a hit, or no hit, situation. Statistics are binary, as in coin tossing. Probabilities of occurrence are determined from the binomial distribution, which measures the numbers of possibilities of occurrence and nonoccurrence in any number of events, given the incidence rate. Specifically, the probability, P, in a random sample of size, N, for n occurrences of DCS and m nonoccurrences, takes the form

$$P(n) = \frac{N!}{n!m!} p^n q^m$$

with

$$n + m = N$$

p the underlying incidence rate (average number of cases of DCS), and q,

$$q = 1 - p$$

the underlying nonincidence. Table 4 lists corresponding binomial decompression probabilities, $P(n)$, for 1% and 10% underlying incidence (99% and 90% nonincidence), yielding 0, 1, and 2 or more cases of DCS. The underlying incidence, p, is the (fractional) average of hits.

As the number of trials increases, the probability of 0 or 1 occurrences drops, while the probability of 2 or more occurrences increases. In the case of five dives, the probability might be as low as 5%, while in the case of 50 dives, the probability could be 39%, both for $p = 0.01$. Clearly, odds even percentages would require testing beyond 50 cases for an underlying incidence near 1%. Only by increasing the number of trials for fixed incidences can the probabilities be increased. Turning that around, a rejection procedure for one or more cases of DCS at the 10% probability level requires many more than 50 dives. If we are willing to lower the confidence of the acceptance, or rejection, procedure, of course, the number of requisite trials drops. Table 4 also shows that the test practice of accepting an exposure schedule following 10 trials without incidence of DCS is suspect, merely because the relative probability of nonincidence is high, near 35%.

Table 4
Probabilities of decompression sickness for underlying incidences

N (dives)	n (hits)	$P(n)$	
		$p = 0.01$, $q = 0.99$	$p = 0.10$, $q = 0.90$
5	0	0.95	0.59
	1	0.04	0.33
	2 or more	0.01	0.08
10	0	0.90	0.35
	1	0.09	0.39
	2 or more	0.01	0.26
20	0	0.82	0.12
	1	0.16	0.27
	2 or more	0.02	0.61
50	0	0.61	0.01
	1	0.31	0.03
	2 or more	0.08	0.96

One constraint usually facing the statistical table designer is a paucity of data, that is, number of trials of a procedure. Data on hundreds of repetitions of a dive profile are virtually nonexistent, excepting bounce diving perhaps. As seen, some 30–50 trials are requisite to ascertain procedure safety at the 10% level. But 30–50 trials is probably asking too much, is too expensive, or generally prohibitive. In that case, the designer may try to employ global statistical measures linked to models in a more complex trial space, rather than a single profile trial space. Integrals of risk parameters, such as bubble number, supersaturation, separated phase, etc., over exposures in time, can be defined as probability measures for incidence of DCS, and maximum likelihood methods used to extract appropriate constants. Such an approach has been developed by Weathersby [32,61,62], plus Vann and Gerth [54–56], and we adopt it for this analysis.

The likelihood of binomial outcome, Φ, of N trials is the product of individual measures of the form

$$\Phi(n) = p^n q^m = p^n (1 - p)^m$$

given n cases of DCS and m cases without DCS, and,

$$n + m = N.$$

The natural logarithm of the likelihood, Ψ, is easier to use in applications, and takes the form

$$\Psi = \ln \Phi = n \ln p + m \ln(1 - p)$$

and is maximized when,

$$\frac{\partial \Psi}{\partial p} = 0.$$

The multivalued probability functions, $p(x)$, generalize in the maximization process according to

$$\frac{\partial \Psi}{\partial p} = \sum_{k=1}^{K} \frac{\partial \Psi}{\partial x_k} \frac{\partial x_k}{\partial p} = 0$$

satisfied when,

$$\frac{\partial \Psi}{\partial x_k} = 0 \quad \text{for } k = 1, K.$$

In application, such constraints are most easily solved on computers, with analytical or numerical methods. For RGBM analysis, the likelihood, Ψ, is typically a function of 2–3 parameters over the whole set of profiles. This requires an extensive computing power coupled to sophisticated numerical techniques and software.

In dealing with a large number of decompression procedures, spanning significant range in depth, time, and environmental factors, an integrated approach to maximum likelihood and risk is necessary. Integral measures, $p(x,t)$ and $q(x,t)$, can be defined over assumed decompression risk, $\zeta(x,t)$,

$$p(x,t) = 1 - \exp\left[-\int_0^t \zeta(x,t')\,dt'\right]$$

and,

$$q(x,t) = \exp\left[-\int_0^t \zeta(x,t')\,dt'\right]$$

with t' any convenient time scale, and ζ any assumed risk, such as bubble number, saturation, venous emboli count, etc. Employing $p(x,t)$ and $q(x,t)$ in the likelihood function, and then maximizing according to the data, permits maximum likelihood estimation of $\zeta(x,t)$. Such an approach can be employed in decompression table fabrication, yielding good statistical estimates on risk as a function of exposure factors.

6. RGBM data correlations and risk estimates

A global statistical approach to table fabrication consists of following a risk measure, or factor p, throughout and after sets of exposures, tallying the incidence of DCS, and then applying maximum likelihood to the risk integral in time, extracting any set of risk constants optimally over all dives in the maximization procedure. In analyzing saturation air and helium data, Weathersby [62] assigned risk as the difference between tissue tension and ambient pressure. One tissue was assumed, with time constant fixed by the data in ensuing maximum likelihood analysis. Another suggested measure of nonincidence, q, is the exponential of risk integrated over exposure time, for every compartment, τ,

$$q(\kappa,\tau) = \exp\left[-\int_0^\infty \zeta(\kappa,\tau,t)\,dt\right]$$

$$\zeta(\kappa,\tau,t) = \kappa[\Pi(\tau,t) - P]$$

with κ a constant determined in the likelihood maximization, P ambient pressure, and $\Pi(\tau,t)$ the instantaneous total tension for tissue with halftime, τ, corresponding to arbitrary tissue compartments for the exposure data. More complex likelihood functions can also be employed, for instance, excess bubble risk according to the varying permeability model of Yount [16,58],

$$\zeta(\mu,\alpha,\tau,t) = \mu\Lambda(t)G(\tau,t),$$

$$\Lambda(t) = [1 - \alpha\varepsilon(t)]$$

with Λ the permissible bubble excess, ε the excitation radius, G the bubble diffusion gradient (dissolved-free gas), and μ and α constants determined in the fit maximization of the data. Another risk possibility is the tissue ratio of Vann and Gerth [56],

$$\zeta(\kappa,\tau,t) = \kappa\left[\frac{\Pi(\tau,t)}{P}\right]$$

Table 5
Nonstop time limits for 1% and 5% DCS probability

Depth d (fsw)	Nonstop limit t_n (min)		
	$p = 0.05$	$p = 0.01$	US Navy
30	240	170	
40	170	100	200
50	120	70	100
60	80	40	60
70	80	25	50
80	60	15	40
90	50	10	30
100	50	8	25
110	40	5	20
120	40	5	15
130	30	5	10

a measure of interest in altitude diving applications. An excited seed volume risk function, suggested by Wienke [60,65], is given by

$$\zeta(\gamma,\beta,\tau,t) = \gamma\Lambda(t)G(\tau,t)$$

$$\Lambda(t) = 4\pi\int_{\varepsilon(t)}^\infty \exp(-\beta r)r^2\,dr$$

with γ and β minimization constants. In the following RGBM analysis, we will use variants of the above.

Hundreds of air dives were analyzed using this procedure, permitting construction of decompression schedules with 95% and 99% nonincidence (5% and 1% bends incidence). Tables were published by US Navy investigators [59,61], and Table 5 tabulates the corresponding nonstop time limits ($p = 0.05, 0.01$), and also includes the standard US Navy (Workman) limits [42,43,57,81,84,85] for comparison. Later reevaluations of the standard set of nonstop time limits estimate a probability rate of 1.25% for the limits. In practice, incidence rates are below 0.001%, and most divers do not dive to the limits.

6.1. RGBM single and repetitive air dive risks

To perform risk analysis with the RGBM Data Bank, a risk estimator need be selected. For diving, dissolved gas and phase estimators are useful. Two, detailed earlier, are extended here. First is the dissolved gas supersaturation ratio, historically coupled to Haldane models, ρ, written in modified ratio form

$$\rho(\kappa,\lambda,t) = \kappa\left[\frac{\Pi(t) - P(t)}{P(t)}\right] - \kappa\exp(-\lambda t)$$

and second, ψ, is the separated bubble volume, invoked by dual phase models,

$$\psi(\gamma,\mu,t) = \gamma\left[\frac{\dot{\phi}(t)}{\phi_i(t)}\right] - \gamma\exp(-\mu t)$$

with $\dot{\phi}$ the bubble volume inflation rate due to excitation, diffusion, and Boyle expansion–contraction (as detailed), and ϕ_i

the initial bubble excitation volume. The exponential terms in both risk functions merely insure data smoothing at early time, that is, as $t \to 0$, then $r \to 0$, too. At late times, $t \to \infty$, the exponential terms vanish. Both risk functions vary in time, exposure, and staging. For simplicity, the asymptotic exposure limit is used in the likelihood integrals for both risk functions, r, across all compartments, τ,

$$1 - r(\kappa, \lambda) = \exp\left[-\int_0^\infty \rho(\kappa, \lambda, t)\,\mathrm{d}t\right],$$

$$1 - r(\gamma, \mu) = \exp\left[-\int_0^\infty \psi(\gamma, \mu, t)\,\mathrm{d}t\right]$$

with *hit–no hit*, likelihood function, Ω, of form

$$\Omega = \prod_{k=1}^{K} \Omega_k$$

$$\Omega_k = r_k^{\delta_k}(1 - r_k)^{1-\delta_k}$$

and logarithmic reduction, Ψ,

$$\Psi = \ln \Omega,$$

where $\delta_k = 0$ if DCS does not occur in profile, k, or, $\delta_k = 1$ if DCS does occur in profile, k. To estimate κ, λ, γ, and μ in maximum likelihood, a modified Levenberg–Marquardt [76,77] algorithm is employed (SNLSE, Common Los Alamos Applied Mathematical Software Library) [78], a nonlinear least squares (NLLS) data fit to an arbitrary logarithmic function (minimization of variance over K data point), with $L2$ error norm. Appendix A details the numerical process, and the same approach is used in all of the following. The likelihood maximization technique amounts to numerically determining κ, γ, λ, and μ according to

$$\frac{\partial \Psi}{\partial r} = \frac{\partial \Psi}{\partial \kappa}\frac{\partial \kappa}{\partial r} + \frac{\partial \Psi}{\partial \lambda}\frac{\partial \lambda}{\partial r} = 0$$

for the dissolved gas ratio estimator, ρ, and,

$$\frac{\partial \Psi}{\partial r} = \frac{\partial \Psi}{\partial \gamma}\frac{\partial \gamma}{\partial r} + \frac{\partial \Psi}{\partial \mu}\frac{\partial \mu}{\partial r} = 0$$

for the phase estimator, ψ.

We assign numerical tasks to processors on the LANL Blue Mountain Machine, a massively parallel processor (MPP) with 2000 nodes according to:

1. each tissue compartment, τ, then, within each compartment;
2. only nitrox data points;
3. only trimix data points;
4. only heliox data points;
5. both nitrox and trimix data points;
6. both nitrox and helium data points;
7. both heliox and trimix data points;
8. all heliox, nitrox, and trimix data points.

Estimating κ, λ, γ, and μ across all domains. The last case, all data, is the full set employed in risk analysis, but there was not much difference in the estimators, seen in mean variance across the partitioned data structures. For 11 tissue compartments and 7 data sets, 77 risk estimates emerge. Only maximum tissue risks are finally averaged and variance computed. In diver staging, certain tissue compartments control the exposure, This is true within dissolved gas algorithms, as well as bubble algorithms. Finally, we find across the partitioned data structures, 2–8 above:

$$\kappa = 0.91 \pm 0.14,$$

$$\lambda = 0.28 \pm 0.11\,\mathrm{min}^{-1}$$

and similarly,

$$\gamma = 0.09 \pm 0.03,$$

$$\mu = 0.88 \pm 0.46\,\mathrm{min}^{-1}$$

as the likelihood fit constants. These can now be used in the risk functions to estimate profile risks. Variances track roughly as the square root of the means, thus means appear Poisson-like in distribution.

As a variance reduction technique, using a random number generator for profiles across 2000 parallel SMP (Origin 2000) processors at LANL, we construct 2,000 subsets, with $K = 750$ across $p \leqslant 0.0067$, for separate likelihood and risk analysis, weighting each processor κ, λ, γ, and μ by the number of sample hits divided by total data set hits. This cuts run and analysis time, plus numerical roundoff errors implicit to likelihood analysis for small r, and large K. The technique also reduces variance across sample means in the analysis. Samples without hits do not score, and are passed up in the process, speeding up run times. The sorting continues through all possible profile combinations, χ, roughly,

$$\chi = \frac{2823!}{750!2073!}.$$

This sampling–weighting technique has proven useful in Monte Carlo applications for particle transport, hydrodynamics, and plasma dynamics. It is similar to importance sampling and variance reduction techniques collectively termed roulette, splitting, biasing, importance weighting, etc. We find good agreement between the canonical estimates above, and the sampling–weighting estimates, that is, the sampling technique yields:

$$\kappa = 0.85 \pm 0.08,$$

$$\lambda = 0.19 \pm 0.05\,\mathrm{min}^{-1}$$

and similarly,

$$\gamma = 0.08 \pm 0.01,$$

$$\mu = 0.92 \pm 0.25\,\mathrm{min}^{-1}$$

noting the variance reduction (error estimates in fits). This technique also resembles condensed-history Monte Carlo, in that only *important* samples are used in the estimation process.

Table 6
Risk estimates for standard air NDLs

d (fsw)	USN NDL t_n (min)	Risk $r(\gamma, \mu)$ (%)	PADI NDL t_n (min)	Risk $r(\gamma, \mu)$ (%)	NAUI NDL t_n (min)	Risk $r(\gamma, \mu)$ (%)	ZHL NDL t_n (min)	Risk $r(\gamma, \mu)$ (%)
35	310	4.3	205	2.0			181	1.3
40	200	3.1	140	1.5	130	1.4	137	1.5
50	100	2.1	80	1.1	80	1.1	80	1.1
60	60	1.7	55	1.4	55	1.4	57	1.5
70	50	2.0	40	1.2	45	1.3	40	1.2
80	40	2.1	30	1.3	35	1.5	30	1.3
90	30	2.1	25	1.5	25	1.5	24	1.4
100	25	2.1	20	1.3	22	1.4	19	1.2
110	20	2.2	13	1.1	15	1.2	16	1.3
120	15	2.0	13	1.3	12	1.2	13	1.3
130	10	1.7	10	1.7	8	1.3	10	1.7

Table 7
Dissolved and separated phase risk estimates for nominal profiles

Profile (depth/time)	Descent rate (msw/min)	Ascent rate (msw/min)	Safety stop (depth/time)	Risk $r(\gamma, \mu)$	Risk $r(\kappa, \lambda)$
14 msw/38 min	18	9	5 msw/3 min	0.0034	0.0062
19 msw/38 min	18	9	5 msw/3 min	0.0095	0.0110
28 msw/32 min	18	9		0.0200	0.0213
37 msw/17 min	18	9	5 msw/3 min	0.0165	0.0151
18 msw/31 min	18	9	5 msw/3 min	0.0063	0.0072
	18	9		0.0088	0.0084
	18	18		0.0101	0.0135
	18	18	5 msw/3 min	0.0069	0.0084
17 msw/32 min	18	9	5 msw/3 min		
SI 176 min					
13 msw/37 min	18	9	5 msw/3 min		
SI 174 min					
23 msw/17 min	18	18	5 msw/3 min	0.0127	0.0232

For recreational diving, all estimators are roughly equivalent, because little dissolved gas has likely separated into free phases (bubbles). Analysis shows this true for all cases examined. The only case where dissolved gas and phase estimators differ (slightly here) is within repetitive diving profiles. The dissolved gas estimator cues on gas buildup in the slow tissue compartments (staircasing for repeats within an hour or two), while the phase estimator cues on bubble gas diffusion in the fast compartments (dropping rapidly over hour time spans). This holding true within all recreational diving distributions, we proceed to the risk analysis.

Nonstop limits (NDLs), denoted t_n, from the US Navy, PADI, NAUI, and ZHL (Buhlmann) tables [40,27] provide a set for comparison of relative DCS risk. Listed in Table 6 are the NDLs and corresponding risks for the profile, assuming ascent and descent rates of 60 fsw/min (no safety stops). Dissolved gas and phase estimates vary little for cases, and only the phase estimates are included. *Surface intervals* (SIs) *between dives are time spent at the surface*.

Risks are internally consistent across NDLs at each depth, and agree with the US Navy assessments in Table 5. Greatest underlying risks occur in the USN shallow exposures. The PADI, NAUI, and ZHL risks are all less than 2% for this set, and risks for single DCS incidence are less than 0.02. PADI and NAUI have reported that incidence rates (p) across all exposures are less than 0.001%, so considering their enviable track record of diving safety, our estimates are liberal. ZHL risk estimates track as the PADI and NAUI risks, again, very safely. Estimates were also independently corroborated (Gerth, priv comm, 2001) within USN and DAN data sets at Duke, both in Tables 6 and 7.

Next, the analysis is extended to profiles with varying ascent and descent rates, safety stops, and repetitive sequence [24,64,66]. Table 7 lists nominal profiles (recreational) for various depths, exposure and travel times, and safety stops at 5 msw. Mean DCS estimates, r, are tabulated for both dissolved gas supersaturation ratio (ZHL) and excited bubble volume (RGBM) risk functions, with nominal variance, $r_{\pm} = r \pm 0.004$, across all profiles.

The ZHL (Buhlmann) NDLs and staging regimens are widespread across decompression meters presently, and are good representations for dissolved gas risk analysis. The RGBM is newer, more modern, and is coming online in decometers and associated software. For recreational exposures, the RGBM collapses to a dissolved gas algorithm. This is reflected in the risk estimates above, where estimates for both models differ little [36,39,43,57,75].

Simple comments hold for the analyzed profile risks. The maximum relative risk is 0.0232 for the three dive repetitive sequences according to the dissolved risk estimator. This translates to 2% profile risk, which is comparable to the maximum NDL risk for the PADI, NAUI, and ZHL NDLs. This type of dive profile is common, practiced daily on liveaboards, and benign. According to Gilliam, the absolute incidence rate [65] for this type of diving is less than 0.02%. Again, our analyses overestimate risk. Effects of slower ascent rates and safety stops are seen only at the 0.25–0.5% level in relative surfacing risk. Safety stops at 5 msw for 3 min lower relative risk an average of 0.3%, while reducing the ascent rate from 18 to 9 msw/min reduces relative risk an average of 0.35%. Staging, NDLs, and constraints imposed by decometer algorithms are consistent with acceptable and safe recreational diving protocols. Estimated absolute risk associated across all ZHL NDLs and staging regimens analyzed herein is less than 2.32%, probably much less in actual practice. That is, we use $p = 0.0067$, and much evidence suggests $p < 0.0001$, some 10 times safer.

Implicit in such formulations of risk tables are assumptions that given decompression stress is more likely to produce symptoms if it is sustained in time, and that large numbers of separate events may culminate in the same probability after time integration. Though individual schedule segments may not be replicated enough to offer total statistical validation, categories of predicted safety might be grouped within subsets of corroborating data. For instance, risks on air dives might be estimated from just nitrox data, risks on trimix from just trimix data, risks on heliox just from heliox data, etc. Since the method is general, any model parameter or meaningful index, properly defined, can be applied to decompression data, and the full power of statistical methods employed to quantify overall risk. While powerful, such statistical methods are neither deterministic nor mechanistic, and cannot predict on first principles. But as a means to table fabrication with quoted risk, such approaches offer attractive pathways for analysis.

Questions of what risk is acceptable to the diver vary. Sport and research divers would probably opt for small risk (1% or less), while military and commercial divers might live with higher risk (5%), considering the nearness of medical attention in general. Many factors influence these two populations, but fitness and acclimatization would probably play strategically.

6.2. UW seafood diver air tables

As another application of the RGBM Data Bank to table construction and analysis, we detail a set of tables of interest to the University of Wisconsin (UW), along with estimated risk for various nonstop limits gleaned from the data. These tables havee no groups, and simple rules. Released mixed gas RGBM tables resulted from similar analyses across both the technical and recreational segments. Such tables are certainly useful for a broad spectrum of diving, and are easy to use. In the last section, we relate reported field testing and extensive application of the algorithm across diving spectra.

Table 8
RGBM repetitive risks for air dives

Depth (fsw)	Maximum time (min)			
	5.14%	3.29%	$r(\gamma, \mu)$ 1.37%	
100	24	20	14	Deep stop 60/1 Shallow stop 15/2
80	38	32	24	Deep stop 50/1 Shallow stop 15/2
60	50	42	32	Deep stop 40/1 Shallow stop 15/2
40	130	120	100	Deep stop 30/1 Shallow stop 15/2

Table 8 lists the maximum NDLs for any series of dives (up to 3) with 60 min SIs between dives. Divers need make a deep stop at 1/2 the maximum bottom pressure for 1 min, plus a shallow safety stop in the 15 fsw zone for 2 min. Descent rate is 60 fsw/min, and ascent rate is 30 fsw/min. The NDLs are listed for maximum risk after 3 repetitive dives to the (same) depth indicated, or to a lesser depth.

Tables like these are of interest to Puerto Rican diving fishermen, and fishing sport divers. NAUI uses a variant, detailed next, for training. Technical Training Agencies also employ mixed gas tables for decompression diving, as well as dive planning software, all based on the RGBM algorithm. Some risk estimates of profiles in these RGBM Technical Tables also follow.

6.3. NAUI air and nitrox recreational tables (sea level—10,000 ft)

For comparison, consider similar RGBM Tables employed by NAUI for air and nitrox diver training, sea level up to 10,000 ft. They are basically the same as the Puerto Rican seafood diver tables above, except that successive dives must always be shallower than the previous. Descent and ascent rates are 75 and 30 fsw/min, and SIs are 60 min. At sea level to 2000 ft elevation, three dives may be made in a day on air or nitrox. At elevations above 2000 ft, only two dives are sanctioned. There are nine RGBM tables in all, three for air, three for EAN32, and three for EAN36, ranging in altitude, 0–2000, 2000–6000, and 9000–10,000 ft. In Tables 9a–c, risks are tabulated at the end of the three or two dive sequences, for just three tables (air at 6000–10,000 ft, EAN32 at 2000–6000 ft, and EAN36 at 0–2000 ft). Risks decrease at any elevation as the oxygen fraction increases, while elevation increases risk for any mixture of nitrogen and oxygen. Moving from left to right (first dive through last permitted dive) successive decrements in permissible depths are seen. Safety stops at half the bottom depth are required for 1–2 min, and an additional shallow stop in the 15 fsw zone for 2 min is part of the protocol. The shallow stop mostly serves to control ascent speed. Maximum risk is seen

Table 9
(a) NAUI RGBM air tables (6000–10,000 ft) (maximum risk after dive 2, $r(\gamma, \mu) = 2.36\%$); (b) NAUI RGBM EAN32 tables (2000–6000 ft) (maximum risk after dive 2, $r(\gamma, \mu) = 1.65\%$); (c) NAUI RGBM EAN36 tables (0–2000 ft) (maximum risk after dive 3, $r(\gamma, \mu) = 1.12\%$)

Dive 1		Dive 2	
Depth (fsw)	Time (min)	Depth (fsw)	Time (min)
90	11	60	28
80	15	55	28
70	21	50	40
60	28	45	40
50	40	40	64
40	64	35	64
30	103	30	103

Dive 1		Dive 2	
Depth (fsw)	Time (min)	Depth (fsw)	Time (min)
100	20	65	43
90	26	60	57
80	33	55	57
70	43	50	84
60	57	45	84
50	84	40	120
40	120	35	120
30	150	30	150

Dive 1		Dive 2		Dive 3	
Depth (fsw)	Time (min)	Depth (fsw)	Time (min)	Depth (fsw)	Time (min)
110	31	80	60	50	150
100	35	75	60	50	150
90	46	70	85	50	150
80	60	65	85	50	150
70	85	60	115	50	150
60	115	55	115	50	150
50	150	50	150	50	150

Table 10
Helitrox NDLs and risk

Depth d (fsw)	Time t_n (min)	Risk $r(\gamma, \mu)$ (%)
70	35	1.4
80	25	1.4
90	20	1.4
100	15	1.4
110	10	1.5
120	8	1.5
130	6	1.4
140	5	1.5
150	4	1.6

in the air tables at 10,000 ft elevation, and minimum risk in the EAN36 tables at sea level.

6.4. Helitrox nonstop limits (NDLs)

Helitrox is enriched trimix, that is, the oxygen fraction is above 21% in the breathing mixture. Helitrox is gaining in popularity over nitrox when helium is available for gas mixing.

Table 11
Comparative helium and nitrogen gas switches

Depth (fsw)	Stop time (min)	
	$r(\gamma, \mu)$ 6.42%	$r(\gamma, \mu)$ 6.97%
	10/65/25 trimix	10/65/25 trimix
400	10.0	10.0
260	1.5	1.5
250	1.0	1.0
240	1.0	1.0
	18/50/32 trimix	18/50/32 trimix
230	0.5	0.5
220	0.5	0.5
210	0.5	0.5
200	0.5	0.5
190	1.0	1.0
180	1.5	1.5
170	1.5	1.0
160	1.5	1.5
150	1.5	2.0
140	2.0	1.5
130	2.0	2.5
120	4.0	4.0
110	4.5	4.0
	40/20/40 trimix	EAN40
100	2.5	2.0
90	2.5	2.0
80	2.5	2.0
70	5.0	4.0
60	6.5	5.5
50	8.0	6.5
40	9.5	7.5
	EAN80	EAN80
30	10.5	10.5
20	14.0	14.0
10	21.0	20.5
	——	——
	123.0	116.0

Diving agencies often use helitrox in the beginning sequence of technical diver training. Listed below in Table 10 are nonstop time limits and corresponding risks, $r(\gamma, \mu)$, for exposures at that depth–time. The mixture is helitrox (enriched 26/17 trimix), sometimes called triox.

These NDL triox risks track closely with NDL risks for air and nitrox.

6.5. Comparative helium and nitrogen staging and risk

Consider a deep trimix dive with multiple switches on the way up. This is a risky technical dive, performed only by seasoned professionals. Table 11 contrasts stop times for two gas choices at the 100 fsw switch. The dive is a short 10 min at 400 fsw on 10/65/25 trimix, with switches at 235, 100, and 30 fsw. Descent and ascent rates are 75 and 25 fsw/min. Obviously, there are many other choices for switch depths, mixtures, and strategies. In this comparison, the oxygen fractions were the same in all mixes, at all switches. Differences between a nitrogen or a helium based decompression strategy, even for

Table 12
WKPP extreme trimix dives

Depth (fsw)	Stop time (min)	Mixture
270	360	10.5/50 trimix
260	1	
250	1	
240	1	18/40 trimix
230	2	
220	2	
210	2	
200	3	
190	3	
180	3	21/35 trimix
170	4	
160	4	
150	5	
140	5	
130	6	
120	7	35/25 trimix
110	8	
100	9	
90	10	
80	12	
70	16	50/16 trimix
60	21	
50	27	
60	34	
50	41	
40	49	
30	150	Pure O_2

Surfacing risk, $r(\gamma, \mu) = 16.67\%$.

this short exposure, are nominal. Such usually is the case when oxygen fraction is held constant in helium or nitrogen mixes at the switch.

Comparative profile reports suggest that riding helium to the 70 fsw level with a switch to EAN50 is good strategy, one that couples the benefits of well being on helium with minimal decompression time and stress following isobaric switch to nitrogen. Shallower switches to enriched air also work, with only a nominal increase in overall decompression time, but with deeper switches off helium to nitrox a source of ICD issues that might best be avoided. Note the risk, $r(\gamma, \mu)$, for the helium strategy, 40/20/40 trimix at 100 fsw, is slightly safer than the nitrogen strategy, EAN40 at 100 fsw, but in either case, the risk is high.

6.6. *WKPP extreme exploration dives*

The WKPP has reported a number of 300 fsw dives with OC and RB systems on trimix for many hours bottom time, and some 8 h of decompression. Pure oxygen is employed in the 30 fsw zone with the help of an underwater habitat. Successful regimens systematically roll back the helium fraction and increase the oxygen fraction in roughly the same proportions, thus maintaining nitrogen fractions low and fairly constant. Diving starts in the cave systems of Wakulla Springs in Florida. Table 12 summarizes the ascent and decompression profile. The risk is, of course, high, but WKPP professionals continue to attempt and complete such extreme exposures, pushing the exploration envelope. These dives served as calibration points for the RGBM algorithm on whole.

Table 13
Trimix dive to 1040 fsw and risk

Depth range (fsw)	Stop range (min)	Mixture
1040	1	5/67 trimix
740–530	0.5–1.5	
520–300	2.0–3.5	
290–180	4.0–6.5	14/56 trimix
170–140	7.0–9.5	
130–70	10.0–15.0	27/56 trimix
60–40	16.0–20.50	
30–20	24.5	80/20 nitrox
10	31.0	Pure O_2

Surfacing risk, $r(\gamma, \mu) = 29.24\%$.

Table 14
Extreme RB dive and risk

Depth (fsw)	Time (min)
444	15.0
290	0.5
280	0.5
270	0.5
260	0.5
250	0.5
240	0.5
230	1.0
220	1.0
210	1.0
200	1.0
190	1.5
180	1.5
170	1.5
160	1.5
150	2.0
140	2.0
130	2.0
120	2.5
110	3.0
100	3.5
90	4.0
80	4.5
70	5.0
60	7.0
50	7.5
40	8.0
30	12.5
20	14.0
10	18.5

Surfacing risk $r(\gamma, \mu) = 5.79\%$.

6.7. *World record OC trimix dive*

Consider risk after an OC dive to 1040 fsw on trimix, with matched ICD switches maintaining the relative fraction of nitrogen constant as helium is reduced in the same measure as oxygen is increased. Dives without this rather well-known strategy ended in some serious hyperbaric chamber time for treatment of vestibular DCS. Reports hint this dive was attempted, maybe accomplished, but contradictions abound. We merely treat it as academic exercise for risk prediction.

Table 15
USS Perry RB repetitive decompression dives and risk

Depth (fsw)	Time (min)
260	40
170	1
160	1
150	1
140	1
130	1
120	1
100	2
90	2
80	2
70	3
60	3
50	4
40	5
30	6
20	9
10	12
0	270
210	20
90	1
80	1
70	1
60	1
50	2
40	2
20	4
10	5

Surfacing risk after dive 1, $r(\gamma, \mu) = 7.48\%$; surfacing risk after dive 2, $r(\gamma, \mu) = 7.79\%$.

Table 13 roughly summarizes the RGBM profile and ascent protocol. Stops range from 740 to 10 fsw for times ranging 0.5 to 31.0 min. Descent rate is assumed to be 60 fsw/min, and ascent rate between stages is assumed to be 30 fsw/min. Mixes and switch depths are indicated, as in Table 12.

The computed risk for this dive is very high, near 30%. Total decompression time is near 415 min. Logistics for stage cylinders are beyond formidable, and the risk for deep support divers is also high.

6.8. Extreme RB profile

Table 14 is a deep RB dive downloaded off the HydroSpace EXPLORER computer. From a number of corners, reports of 400 fsw dives on RB systems are becoming commonplace. Consider this one to 444 fsw for 15 min. Diluent is 10/85 trimix, and pp_{O_2} setpoint is 1.1 atm. From a decompression standpoint, RB systems are the quickest and most efficient systems for underwater activities. The higher the pp_{O_2}, the shorter the overall decompression time. That advantage, however, needs to be played off against increasing risks of oxygen toxicity as oxygen partial pressures increase, especially above 1.4 atm. The higher percentage of oxygen and lower percentage of inert gases in higher pp_{O_2} setpoints of closed circuit rebreathers (CCRs) results in reduced risks, simply because gas loadings and bubble couplings are less in magnitude and importance. This shows up in any set of comparative pp_{O_2} RB calculations, as well as in OC versus RB risk estimates (Table 14).

The risk associated with this 400 fsw is less than a similar dive on trimix to roughly the same depth for a shorter period of time, that is, Table 11.

6.9. USS Perry deep RB wreck dives

A team of divers uncovered the wreck of the USS Perry in approximately 250 fsw off Anguar, and explored it for a week on RBs. Diving in extremely hazardous and changing currents, their repetitive decompression profile appears in Table 15. Profiles and risk for the two dives, separated by 4 h SI, are nominal, with no accounting of exertion effort in current implied. Diluent is 10/50 trimix, with a pp_{O_2} setpoint of 1.3 atm.

7. Summary

The LANL reduced gradient bubble model (RGBM) has been detailed, including correlations and data linkage within the LANL Data Bank. The Bank stores technical, mixed gas diving profiles with outcomes. Some 2800 + deep stop profiles reside within the Bank, with 19 cases of DCS. Parameters within the RGBM have been extracted from the RGBM Data Bank using maximum likelihood techniques, and a Monte Carlo-like sampling technique was also described and used to accelerate likelihood analysis. Risk estimates for some select NDLs, tables, meter algorithms, and diver profiles in the RGBM Data Bank were tabulated, using a bubble phase volume estimator integrated over the whole profile.

A few important points can be reiterated here:

1. Deep stop data is intrinsically different from data collected in the past for diving validation, in that previous data is mainly based on shallow stop diver staging, a bias in model correlations.
2. Deep stop data and shallow stop data yield the same risk estimates for nominal, shallow, and nonstop diving because bubble models and dissolved gas models converge in the limit of very small phase separation.
3. If shallow stop data is employed in all the cases detailed, dissolved gas (only) risk estimates will be categorically higher than those computed herein.
4. Data entry in the RGBM Data Bank is a ongoing process of profile addition, extended exposure-depth range, and mixed gas diving application.

Appendix A. Software, data analysis, and field data

A broad overview of software, numerical techniques, and field data follows.

A.1. Staging and risk software

A rundown of the LANL (data correlated) software configuration of the RGBM used in analyses is tabulated. The package is under constant refinement and updating, and can be used on OC or RB systems. It has been a mainstay in dive planning and operations here at LANL. Parameters in the model and software have been calibrated against profile outcomes in the RGBM Data Bank. The same module is used to generate bubble and dissolved gas risk functions employed in likelihood analysis of data.

1. *Module*: integrated bubble excitation, dissolved gas and bubble gas transfer, material EOS for surfactants, Boyle expansion and contraction, and staging routines, with waypoints prior to ascent, for nitrox, heliox, and trimix.
2. *Source code*: 1640 lines.
3. *Language/compiler*: FORTRAN 77/90, BASIC.
4. *CRAY YMP running time*: 1 s for deep trimix profile with five gas switches on way up.
5. *Input*: altitude, bottom mixture/pp_{O_2}, ascent/descent rate, switch levels and gas mixtures/pp_{O_2}s, predive breathing gas, safety knobs, and previous dive history.
6. *Output*: controlling tissue compartments, stop depth and times, supersaturation gradient, permissible supersaturation, effective bubble and gas parameters, critical phase volume, and dive profile.

Commercial versions are marketed by GAP, ABYSS, and HydroSpace Engineering. Meter implementations are marketed by Suunto, Mares, HydroSpace, Dacor, Plexus, Zeagle, Steam Machines, UTC, and others in the works.

A.2. Statistical analysis and risk

To estimate κ, λ, γ, and μ in maximum likelihood, a modified Levenberg–Marquardt algorithm is employed (*SNLSE*, Common Los Alamos Applied Mathematical Software Library), an NLLS data fit to an arbitrary logarithmic function (minimization of variance over K data points here), with computed $L2$ error norm. The mathematical approach is well known. To estimate a function Φ, using a fit set, Υ, that is,

$$\Phi = \frac{1}{2}\sum_{m=1}^{M}[\Upsilon_m(x_m)]^2$$

or, in vector notation,

$$|\mathbf{\Phi}| = \tfrac{1}{2}\mathbf{\Upsilon}(x)\cdot\mathbf{\Upsilon}(x)$$

a solution vector, $\mathbf{p}$, is found satisfying,

$$[\mathbf{J}^{\dagger}\mathbf{J} + \chi\mathbf{I}]\mathbf{p} = -\mathbf{J}^{\dagger}\mathbf{f}$$

with $\mathbf{J}$ the Jacobian (derivative determinant) of $\mathbf{\Upsilon}$,

$$\mathbf{J} = \frac{\partial\mathbf{\Upsilon}}{\partial\mathbf{x}}.$$

$\mathbf{J}^{\dagger}$ the hermitian inverse (transpose) of $\mathbf{J}$, and $\mathbf{I}$ the identity operator. The χ are positive constants, and $\mathbf{p}$ is the approximation to $\mathbf{\Phi}$. Numerically, all Jacobian derivatives are estimated and used in the minimization fit. Functions are generally nonlinear in form and behavior, and the error is L2 (variance in fit to exact values). The process is iterative, with each update, $\mathbf{q}$, of $\mathbf{p}$, obtained from the Jacobian differential expansion

$$\mathbf{\Upsilon}(\mathbf{p}+\mathbf{q}) = \mathbf{\Upsilon}(\mathbf{p}) + \mathbf{J}\mathbf{q}.$$

A.3. Numerical sampling technique

The enormous computing power and lightning speed of the LANL Blue Mountain MPP (massively parallel processor) permits fast and compute intensive numerical experiments with data. So as a variance reduction technique across the full canonical data set, using a random number generator for profiles across 2000 parallel SMP (Origin 2000) processors at LANL, we construct 2000 subsets, with $K = 750$ across $p \leqslant 0.0067$, for separate likelihood analysis, weighting each processor κ, λ, γ, and μ by the number of sample hits divided by the number of population hits. This cuts run and analysis time, plus numerical roundoff errors implicit to likelihood analysis for small r, and large K. The sorting continues through all possible profile combinations, χ, roughly,

$$\chi = \frac{2823!}{750!2073!},$$

which is a very large set of calculational samples for any computer, save massively parallel, very fast, large core machines available at select locations in the world. Processors with zero DCS hits in the sample contribute nothing to the total tally. Such a weighting technique has tremendous advantages in Monte Carlo applications, providing fast and reliable estimates of statistical quantities over condensed event space. At LANL, major gains are seen in particle transport, hydrodynamic, and plasma applications of Monte Carlo techniques. The method is similar to roulette, biasing, importance sampling, splitting, and other variance reduction techniques utilized in transport phenomenology. Recall that the Blue Mountain MPP boasts overall processor speeds in the teraflop range (10^{12} binary operations/s). The massively compute intensive program above takes some 30–40 s.

A.4. Field data in LANL data bank

Models need validation and field testing. Often, strict hyperbaric chamber tests are not possible, economically nor otherwise, and models employ a number of benchmarks and regimens to underscore viability. The following are some supporting the RGBM phase model and (released) nitrox, heliox, and trimix diving tables and meters. Profiles are recorded in the RGBM Data Bank, and are representative of entries in terms of dive counts and technical diving applications.

1. Counterterror and countermeasures Team (C&C) RB and OC exercises have used the RGBM (iterative deep stop version) for a number of years, logging some 2245 dives on mixed gases (trimix, heliox, and nitrox) with 0.4% incidence of DCS—85% were deco dives, and 55% were repeats with at least 2 h SIs, with most in the forward direction (deepest dives first). Some nine cases of DCS were logged

by the Team, mainly in the deep reverse profile category on nitrox and trimix, plus RB hits on heliox.

2. NAUI Technical Diving has been diving the deep stop version for the past 9 years, some estimated 22,000 dives, on mixed gases down to 300 fsw, with two reported cases of DCS, both on trimix. Some 15 divers, late 1999, in France used the RGBM to make two mixed gas dives a day, without mishap, in cold water and rough seas. Same thing in the warm waters of Roatan in 2000 and 2001.
3. NAUI Worldwide released a set of RGBM Tables for air, EAN32, and EAN36 recreational diving, from sea level to 10,000 ft, a few years ago. Minimum SIs of 1 h are supported for repetitive diving in all tables, and safety stops for 2 min in the 15 fsw zone, plus 1 min deep stops at half bottom depth, are required always. Tables were tested by NAUI Insructor Trainers, Instructors, and Divemasters over a 2 year period without mishap, and continue so today as the mainstay teaching Tables in NAUI basic air and nitrox courses.
4. Modified RGBM recreational algorithms (Haldane imbedded with bubble reduction factors limiting reverse profile, repetitive, and multiday diving), as coded in Suunto, Mares, Dacor, UTC, Zeagle, Steam Machines, GAP, ABYSS, HydroSpace, Plexus decometers, maintain an already low DCS incidence rate of approximately 1/50,000 or less. More RGBM decompression meters, including mixed gases, are in the works.
5. A cadre of divers and instructors in mountainous New Mexico, Utah, and Colorado have been diving the modified RGBM at altitude, an estimated 1200 dives, without peril. Again, not surprising since the altitude RGBM is slightly more conservative than the usual Cross correction used routinely up to about 8000 ft elevation, and with estimated DCS incidence less than 1/10,000.
6. Within decometer implementations of the RGBM, only a few scattered DCS hits have been reported in nonstop and multidiving categories, beyond 1,300,000 dives or more, up to now, according to statistics furnished the author (B.R.W) by meter vendors.
7. Extreme hyperbaric chamber tests for mixed gas RGBM protocols are in the works, and less stressful exposures will be addressed also—extreme here means 300 fsw and beyond.
8. As seen, probabilistic decompression analysis of selected recreational air RGBM profiles, calibrated against similar calculations of the same profiles by Duke, help validate the RGBM on computational bases, suggesting the RGBM has no more theoretical risk than other bubble or dissolved gas models (Weathersby, Vann, Gerth methodology at USN and Duke).
9. All divers and Instructors using RGBM decometers, tables, or Internet software have been asked to report individual profiles to DAN Project Dive Exploration (Vann, Denoble at Duke), plus to the RGBM Data Bank (Wienke, O'Leary at LANL and NAUI).
10. GAP, HydroSpace RGBM Simulator, and ABYSS are NET software packages that offer the modified RGBM (folded Buhlmann ZHL) and, especially, the full up, deep stop version for any gas mixture, have a fairly large contingent of tech divers already using the RGBM and have not received any reports of DCS to date. The EXPLORER RGBM Simulator is furnished to meter owners of the HydroSpace EXPLORER.
11. Extreme WKPP profiles in the 300 fsw range on trimix were used to calibrate the RGBM. WKPP profiles are the most impressive application of RGBM staging, with as much as 12 h less decompression time for WKPP helium based diving on RGBM schedules versus Haldane schedules, with estimated 200 dives.
12. Ellyat, a TDI Instructor, dived the Baden in the North Sea to 540 fsw on RGBM Tables on two different occasions, and 3 h were shaved off conventional hang time by RGBM application. Unfortunately, with diver error and mismatched gas switching strategies from helium to nitrogen, dives to 840 fsw resulted in vestibular DCS.
13. NAUI Worldwide released sets of deep stop RGBM nitrox, heliox, and trimix technical and recreational Tables that have been tested by NAUI Technical Diving Operations over the past 9 years, with success and no reported cases of DCS, for OC regulators and RBs.
14. Doppler and imaging tests in the laboratory, and analyses by Bennett, Marroni, Brubakk and Wienke, and Neuman all suggest reduction in free phase counts with deep stop staging.
15. Deep air RGBM Tables with surface oxygen decompression are employed by American oil patch diving companies.
16. Scorese, a NAUI instructor, and his students made a total of 234 dives on the Andrea Doria using RBs and RGBM (constant pp_{O_2}) RB tables, and various nitrogen and trimix diluents. Dive abortions off RBs employed ranged RGBM (OC) tables as bailouts, and witnessed no mishaps.
17. Freauf, a Navy SEAL in Hawaii, logged 20 trimix decompression dives beyond 250 fsw on consecutive days using RGBM Tables (pure oxygen switch at 20 fsw).
18. NEDU is testing to see if deep stops are necessary and cost effective for air and nitrox Navy divers, that is, risk versus decompression time.
19. Melton, owner of HydroSpace Engineering and developer of the RGBM EXPLORER (OC plus RB) dive computer reports hundreds of dives in the 400 fsw range on the RGBM EXPLORER.
20. GAP, Gas Absorption Program, an RGBM software product out of the Netherlands, supports brisk and sustained use of the RGBM within the tec and rec diving community.
21. NEDU in Panama City is performing deep stop man trials in open water using a US Navy bubble model.
22. Heliox RGBM Tables are being used by a commercial diving operation in Argentina.
23. Raine, a wreck diver in California, reports hundreds of RGBM dives in the 250 fsw range with low Doppler counts.
24. The RGBM site, *RGBMdiving.com*, receives hundreds of hits weekly, and provides custom RGBM tables.
25. ANDI, a training agency, has adopted a custom version of GAP for diver training on mixed gases, OC and RBs.

26. NAUI similarly employs a custom version of GAP for dive planning, with nominal GAP parameter settings recovering released and published NAUI RGBM tables.
27. O'Leary, Director NAUI Technical Operations, has made over 70 dives on OC and RB systems using RGBM Table and the HydroSpace EXPLORER to depths beyond 250 fsw, with anywhere from 6 to 9 other divers during NAUI Technical Instructor Training Courses.
28. O'Leary, Sharp, Scorese, Bell, Hunley, and six other NAUI Instructors used RGBM OC and RB tables to dive the USS Perry in Anguar in very strong currents, down to 260 fsw, logging two repetitive deco dives a day for a week or so.
29. The Finnish Diving Federation (FDF) has adopted RGBM tables for recreational air and nitrox diver training, as well as light decompression exposures down to 130 fsw.

With DCS binomially distributed in incidence probability, many trials are needed (or other close profiles) to fully validate any model at the 1% level. Additionally, full validation requires DCS incidences, the higher the number, the better, contrary to desired dive outcomes. While the foregoing list of field tests and profiles are not controlled scientific experiments with attendant data collection, the shear number of diving events and diversity of exposure spectrum ought not be discounted nor treated lightly. Collective information has been dubbed a *living laboratory* by segments of the technical, scientific, and operational diving community.

Conflict of interest statement

None declared.

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